

Primordial black hole evolution in tensor-scalar cosmology

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A perturbative analysis shows that black holes do not remember the value of the scalar field φ at the time they formed if φ changes in tensor-scalar cosmology. Moreover, even when the black hole mass in the Einstein frame is approximately unaffected by the changing of φ , in the Jordan-Fierz frame the mass increases. This mass increase requires a reanalysis of the evaporation of primordial black holes in tensor-scalar cosmology. It also implies that there could have been a significant magnification of the (Jordan-Fierz frame) mass of primordial black holes.

I. INTRODUCTION

Various approaches to unified theories and quantum gravity suggest the possibility of one or more massless scalar fields such as the dilaton or other moduli coupled to the trace of the energy-momentum tensor of matter. Theories with such fields in addition to the spacetime metric are dubbed “tensor-scalar” theories. The action for an illustrative class of such theories with one scalar is [1]

$$S = (16\pi G_*)^{-1} \int d^4x g_*^{1/2} \left(R_* - 2g_*^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right) + S_m[\psi_m, A^2(\varphi)g_{*\mu\nu}]. \quad (1.1)$$

The metric $g_{*\mu\nu}$ is called the “Einstein metric” and R_* is its scalar curvature. The matter fields are collectively denoted ψ_m , and they couple universally to the “Jordan-Fierz metric”

$$\tilde{g}_{\mu\nu} \equiv A^2(\varphi)g_{*\mu\nu}, \quad (1.2)$$

where the form of the coupling function $A(\varphi)$ is input which presumably descends from a more fundamental theory. The Newton constant \tilde{G} as measured in Cavendish type experiments is given by

$$\tilde{G} = (1 + \alpha^2(\varphi))A^2(\varphi)G_*, \quad (1.3)$$

where $\alpha \equiv d \ln A / d\varphi$, which is not in fact constant in a cosmological solution. Quantities defined with respect

to $g_{*\mu\nu}$ are commonly referred to as being given in the “Einstein frame” while those defined with respect to $\tilde{g}_{\mu\nu}$ are said to be in the “Jordan-Fierz frame”.

Damour and Nordtvedt [2] identified a generic attractor mechanism which drives a tensor-scalar cosmology to a purely tensor (Einstein) one at late times, thus significantly increasing the plausibility of these models. According to this mechanism the scalar φ is driven to a local minimum of $A(\varphi)$. Definite predictions for the residual effects of the scalar(s) emerge from their analysis. The Jordan-Fierz-Brans-Dicke theory is a special case of (1.1) with $A(\varphi) = \exp(\alpha_0 \varphi)$. Since such a coupling function has no minimum the attractor mechanism does not work for this theory and fine tuning is therefore required. Generically, however, one might expect $A(\varphi)$ to possess local minima. A related mechanism in a model motivated by string theory was studied by Damour and Polyakov [3].

Barrow and Carr initiated a study of the evolution of a population of primordial black holes in tensor-scalar cosmology [4,5]. They considered two differing scenarios [4], “scenario A” in which the value of the scalar field (and hence \tilde{G}) at the horizon evolves along with the cosmological value, and “scenario B” in which a black hole “remembers” the value of the scalar field at the time of its formation. The black hole evolution is very different in the two cases if, as is natural in these models, Newton’s constant (1.3) decreases over cosmological timescales. Analyzing the black hole evolution in the Jordan-Fierz frame, and assuming the black hole mass \tilde{M} changes only due to the Hawking evaporation, Barrow and Carr argued that in scenario A the Hawking luminosity increases as $L \sim T_H^4 \times (\text{Area}) \sim (\tilde{G}\tilde{M})^{-2}$. A black hole born when \tilde{G} was larger would thus have a longer lifetime than would be surmised from the present value of \tilde{G} . In scenario B on the other hand, the black hole remembers the primordial value of \tilde{G} so its lifetime would be even longer.

It turns out that neither of these two scenarios is correct. It will be shown in this paper first that there is no “gravitational memory”, so the value of \tilde{G} at the black hole keeps up with the cosmological change. Second, even if the mass of the black hole is essentially constant in the Einstein frame, the mass *increases* in the Jordan-Fierz frame in proportion to $1/A(\varphi)$. This mass increase would counteract the Hawking evaporation (as described in the Jordan-Fierz frame), and could significantly “magnify” the mass of primordial black holes.

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II. EVOLUTION OF THE SCALAR FIELD AT THE HORIZON

The problem to solve is this: if a small black hole is embedded in a cosmology with changing scalar field how does the scalar field at the horizon evolve? At the outset, it is worth remarking that it seems unlikely that the scalar field would be pinned at the horizon, since that would entail increasing gradients in the scalar field as it interpolates between the horizon and the changing cosmological value. It would seem that such gradients would lead to propagation that would even out the field.

We approach the problem by exploiting the great separation of scales between the black hole and the cosmological background. Since the black hole is much smaller than the cosmological length or time scales it is reasonable to think of it as sitting in a local asymptotically flat space, with a boundary condition for the scalar field set by the cosmological evolution $\varphi_c(t)$. If the scalar field at the black hole follows $\varphi_c(t)$ then, since this change is very slow compared with the size of the black hole, the solution should be a small perturbation of the stationary black hole. If such a perturbation exists in which φ at the horizon keeps up with $\varphi_c(t)$, then our assumption will be shown to be consistent.

For simplicity we first discuss nonrotating black holes, and we work in the Einstein frame. The only such black holes in Einstein-scalar gravity are the Schwarzschild metric with a constant scalar, and the only spherically symmetric perturbations of these black holes are pure scalar fields satisfying the wave equation in the Schwarzschild background. Thus we need only look for spherically symmetric solutions to the wave equation,

$$\left(-g^{tt}\partial_t^2 + \frac{1}{\sqrt{-g}}\partial_r\sqrt{-g}g^{rr}\partial_r\right)\varphi(t, r) = 0, \quad (2.1)$$

(in Schwarzschild coordinates) subject to the boundary condition

$$\varphi(t, r = \infty) = \varphi_c(t) = \dot{\varphi}_c t. \quad (2.2)$$

Since the cosmological timescale is assumed to be very long compared with that of the black hole, it is consistent to set the cosmological scalar perturbation equal to a linear function of the Schwarzschild time coordinate t as we have done here with $\dot{\varphi}_c$ a constant.

The unique solution to the wave equation (2.1) depending only on t and satisfying the boundary condition is just the boundary value itself,

$$\varphi_1(t, r) = \dot{\varphi}_c t. \quad (2.3)$$

However this can not be the perturbation we are looking for since the coordinate t and hence the “perturbation” φ_1 diverges at the black hole horizon. The unique solution depending only on r is given up to constants by

$$\varphi_2(t, r) = \ln(1 - r_0/r) \quad (2.4)$$

where r_0 is the Schwarzschild radius and we have used $\sqrt{-g}g^{rr} = (r - r_0)r \sin\theta$. This solution also diverges at the horizon, but there is a linear combination of these two solutions that is regular at the horizon.¹ Since φ_2 vanishes at infinity, this linear combination will be the perturbation we seek.

The advanced time coordinate $v = t + r^*$, with $r^* = r + r_0 \ln(r/r_0 - 1)$, is regular on the horizon. In terms of v we have $\ln(1 - r_0/r) = (v - t - r)/r_0 + \ln r_0/r$, hence the linear combination of φ_1 and φ_2 which is regular on the horizon and approaches φ_c at infinity is

$$\varphi_3 = \varphi_1 + r_0 \dot{\varphi}_c \varphi_2 = \dot{\varphi}_c (v - r - r_0 \ln \frac{r}{r_0}). \quad (2.5)$$

The existence of the solution (2.5) establishes the result that the cosmological change of the scalar field can extend smoothly to the black hole horizon. Any other regular perturbation satisfying the boundary condition must be a superposition of waves which will dissipate by falling into the black hole or spreading out to infinity, hence after transients the perturbation (2.5) will describe the secular change of the scalar field.

A surface of constant φ_3 intersects the horizon at an advanced time v_H and reaches infinity at a cosmological or Schwarzschild time t_∞ , these two times being related by $v_H - r_0 = t_\infty$. The two surfaces $v = v_H$ and $t = t_\infty$ thus intersect at $r^* = v_H - t_\infty = r_0$ which is at around $r \simeq 1.5r_0$. Thus there is not much “lag” between the horizon value and the cosmological value.

The solution φ_3 generalizes with little change to the case of a rotating black hole. Using Boyer-Lindquist coordinates for the Kerr metric the wave equation for functions of t and r again takes the form (2.1). Hence we again find that $\varphi_1(t)$ (2.3) solves the wave equation, and in place of (2.4) we find the stationary solution

$$\varphi_{2'} = \ln \frac{r - r_+}{r - r_-} \quad (2.6)$$

where r_\pm are the radii of the outer and inner horizons and we have used $\sqrt{-g}g^{rr} = (r - r_+)(r - r_-) \sin\theta$. The advanced time coordinate $v = t + r^*$, now with $r^* = \int dr(r^2 + a^2)/(r - r_+)(r - r_-) = r + \frac{r_+ + r_-}{r_+ - r_-} [r_+ \ln(\frac{r}{r_+} - 1) - r_- \ln(\frac{r}{r_-} - 1)]$, is regular on the Kerr horizon. The linear combination of φ_1 and $\varphi_{2'}$ which is finite on the horizon and approaches φ_c at infinity is given by

$$\begin{aligned} \varphi_{3'} &= \varphi_1 + \left(\frac{2mr_+}{r_+ - r_-} \dot{\varphi}_c \right) \varphi_{2'} \\ &= \dot{\varphi}_c \left[v - r - 2m \ln\left(\frac{r}{r_-} - 1\right) + \frac{2mr_+}{r_+ - r_-} \ln \frac{r_+}{r_-} \right] \end{aligned} \quad (2.7)$$

where $2m = r_+ + r_-$. Thus also for a rotating black hole there is a perturbation describing a scalar field changing

¹This observation is due to Amos Ori.

linearly with time in the same way at the horizon as at infinity.

III. EVOLUTION OF THE BLACK HOLE MASS

The mass of a black hole may grow by accretion or decrease by emission of Hawking radiation. Aside from standard accretion processes, in tensor-scalar cosmology a changing scalar at the horizon entails a flux of energy into the black hole, thus increasing its mass. In addition, we shall see that even if the mass in the Einstein frame is approximately constant, the mass defined in the Jordan-Fierz frame changes simply due to the conformal scaling of the metric (1.2).

To avoid confusion, in this section asterisk subscripts are attached to quantities defined with respect to the Einstein metric, and tildes adorn quantities defined with respect to the Jordan-Fierz metric. This asterisk is unrelated to the superscript on the radial tortoise coordinate in the previous section.

The rate of change of a black hole mass with respect to cosmological time, arising from the changing scalar, is most easily evaluated in the Einstein frame. Let us consider for simplicity a nonrotating black hole. Provided the change of the mass is slow on the scale of the black hole, i.e. $dM_*/dt_* \ll M_*/r_{*0}$, it is given approximately by the flux of Killing energy across the horizon,

$$dM_*/dt_* = \text{Area}_* \times T_{*v_*v_*}^{(\varphi)} = 2G_*^{-1}r_{*0}^2(d\varphi_c/dt_*)^2. \quad (3.1)$$

If φ changes by an amount $\Delta\varphi$ over an Einstein-frame cosmological time Δt_* , the fractional increase of the mass is given by

$$\frac{\Delta M_*}{M_*} \sim \frac{r_{*0}}{\Delta t_*} (\Delta\varphi)^2 \quad (3.2)$$

provided $r_{*0}\Delta\varphi/\Delta t_* \ll 1$. Small black holes can therefore experience a huge variation in φ with essentially no increase of the mass M_* in the Einstein frame as long as the change is adiabatic. This conclusion also follows directly from the fact that in adiabatic processes the black hole entropy $A_*/4G_* = 4\pi G_* M_*^2$ is unchanged.

The relation between the mass \widetilde{M} in the Jordan-Fierz frame and that in the Einstein frame is given by $\widetilde{M}/M_* = dt_*/d\widetilde{t}$ (since the mass (energy) is by definition the value of the generator of (asymptotic) Killing time translations, which scales inversely with the time coordinate), which from (1.3) yields

$$\widetilde{M} = A^{-1}(\varphi)M_*. \quad (3.3)$$

Thus, even when M_* is constant, the black hole mass in the Jordan-Fierz frame is *not constant*. This conclusion is

somewhat surprising since, by contrast, the rest mass of ordinary matter is constant in the Jordan-Fierz frame.²

In the Einstein frame both the black hole mass and Newton's constant are constant, hence the Hawking evaporation process is identical to that in ordinary Einstein gravity. All that is needed for cosmology then is to transform the results back into the Jordan-Fierz time frame in which the matter is typically understood to evolve.

If alternatively the evaporation is analyzed in the Jordan-Fierz frame, as in [4,5], it is necessary to take into account not only the change of Newton's constant (1.3) but also the change of \widetilde{M} (3.3) arising from the time dependence of $A(\varphi)$. To determine the dependence of the Hawking luminosity \widetilde{L} on \widetilde{G} and \widetilde{M} we must start with the fact that \widetilde{L} is determined directly by geometrical quantities in the Jordan-Fierz frame³: the surface gravity $\widetilde{\kappa}$ (which determines the Hawking temperature $\widetilde{T}_H = \hbar\widetilde{\kappa}/2\pi$) and the black hole absorption coefficients $\widetilde{\Gamma}$. The luminosity behaves approximately as $\widetilde{L} \sim \widetilde{\text{Area}} \times \widetilde{\kappa}^4$ which for a Schwarzschild black hole becomes $\widetilde{L} \sim \widetilde{r}_0^{-2}$ where \widetilde{r}_0 is the Schwarzschild radius. Note however that \widetilde{r}_0 is *not* the same as $2\widetilde{G}\widetilde{M}$. Being purely geometrical, \widetilde{r}_0 must be related to the Schwarzschild radius in the Einstein frame $r_{*0} = 2G_*M_*$ by the same scale factor that relates the two metrics (1.2), $\widetilde{r}_0 = A(\varphi)r_{*0}$. Using (1.3) and (3.3) on the other hand we find $\widetilde{G}\widetilde{M} = (1 + \alpha^2)AG_*M_*$, hence evidently $\widetilde{r}_0 = 2\widetilde{G}\widetilde{M}/(1 + \alpha^2)$.

If $A(\varphi)$ decreases by a large amount between the epoch of primordial black hole formation and today then it is possible that the Jordan-Fierz frame mass of primordial black holes would have been significantly magnified. Such large changes of $A(\varphi)$ are not inconsistent with observations. According to Damour and Pichon [7] nucleosynthesis bounds are consistent with $A_{10\text{ MeV}}/A_{\text{today}}$ as large as 150 or even greater. Going back further to the early radi-

²In the string motivated variation [3] of (1.1), however, one expects dilaton-dependence of (at least) hadron masses in both the Jordan-Fierz and Einstein frames.

³The surface gravities in the Jordan-Fierz and Einstein frames are simply related by $\kappa = A(\varphi_c)\widetilde{\kappa}$ where φ_c is the asymptotic cosmological value of φ . One way to see this [6] is to note that surface gravity is conformally invariant except for the effect due to the different asymptotic normalization of the Killing field. The absorption coefficients would in general not transform in any simple way except for conformally coupled fields, however in the adiabatic approximation the conformal factor $A^2(\varphi)$ relating the two metrics is constant, so in fact the absorption coefficients for massless fields are identical in the two frames, and those for massive fields are related by rescaling the mass. Thus one can also determine the luminosity L_* directly in the Einstein frame. The luminosities \widetilde{L} and L_* are related simply by the transformation of the energy and time scales, hence we have $L_* = A^2(\varphi)\widetilde{L}$.

ation era $A(\varphi)$ could in principle have been tremendously larger. Indeed, Damour and Polyakov argued in a dilatonic model [3] that, as a result of the effect accumulated each time the temperature drops through the annihilation energy of a particle species during the radiation era, $A(\varphi)$ is decreased by a net factor given by $A_{\text{out}}/A_{\text{in}} \sim (A_{\text{out}}/A_{\text{today}})^{-1/F^2}$ where $F = -1.87 \times 10^{-4} \kappa^{-9/4}$ and κ is a parameter which is naturally of order unity. Thus $A(\varphi)$ could have decreased since the beginning of the radiation era by a factor of order 10^{10^8} for example if $A_{\text{out}}/A_{\text{today}} = 100$. Although the quadratic model $\ln A(\varphi) \propto (\varphi - \varphi_{\text{min}})^2$ underlying these calculations should not be taken seriously over such a range, the numbers serve to illustrate the point that $A(\varphi)$ could have been extremely large at the beginning of the radiation era compared to now.

It is thus conceivable that the mass of a small primordial black hole could have been magnified by a very large factor. The mass today depends of course not only on the mass magnification factor but on the initial mass. Since the Einstein frame mass is unchanged and today the two frames coincide, this “mass magnification” would only deserve the name if the expected masses for primordial black holes were fixed in the Jordan-Fierz frame rather than the Einstein frame. Whether this is true would depend on the physics producing these black holes.

The potential mass spectrum of primordial black holes will not be discussed here, except to point out one fact. Carr and Hawking’s original estimate [8], that the mass of primordial black holes formed from collapse of early universe overdensities might be expected to be of order the horizon mass, arose from the agreement $M_{\text{horizon}} \sim M_{\text{Jeans}}$ of the upper limit horizon mass $M_{\text{horizon}} \sim t/G$ and the lower limit Jeans mass $M_{\text{Jeans}} = (G^3 \rho)^{-1/2}$. This coincidence does not necessarily hold in tensor-scalar gravity. Since the Jordan-Fierz and Einstein metrics are conformally related, they define the same particle horizon, so the horizon masses are related by (3.3), $\widetilde{M}_{\text{horizon}} = A^{-1} M_{* \text{horizon}} \sim A^{-1} (G_*^3 \rho_*)^{-1/2}$ (where ρ_* is the Einstein frame cosmological energy density). The Jeans mass is naturally defined in the Jordan-Fierz frame since the matter is minimally coupled there. Hence, using $\rho_* = A^4 \widetilde{\rho}$ and (1.3), we have

$$\begin{aligned} \widetilde{M}_{\text{Jeans}} &= (\widetilde{G}^3 \widetilde{\rho})^{-1/2} \\ &= (1 + \alpha^2)^{-3/2} A^{-1} (G_*^3 \rho_*)^{-1/2} \\ &\sim (1 + \alpha^2)^{-3/2} \widetilde{M}_{\text{Horizon}}. \end{aligned} \quad (3.4)$$

If $\alpha \lesssim 1$ then the upper and lower limits still coincide so the expected mass would be the horizon mass at the time of formation. If however $\alpha \gg 1$, then the Jeans mass is much smaller than the horizon mass, which would allow a much smaller mass for primordial black holes formed from overdensities than otherwise expected in the standard scenario [8].

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- [1] For a review of tensor-scalar theories see T. Damour and G. Esposito-Farèse, *Class. Quant. Grav.* **9**, 2093 (1992).
 - [2] T. Damour and K. Nordtvedt, *Phys. Rev. Lett.* **70**, 2217 (1993); *Phys. Rev. D.* **48**, 3436 (1993).
 - [3] T. Damour and A.M. Polyakov, *Nucl. Phys. B* **423**, 532 (1994); *Gen. Rel. Grav.* **26**, 1171 (1994).
 - [4] J.D. Barrow, *Phys. Rev. D* **46**, R3227 (1992).
 - [5] J.D. Barrow and B.J. Carr, *Phys. Rev. D* **54**, 3920 (1996).
 - [6] T. Jacobson and G. Kang, *Class. Quantum Grav.* **10**, L201 (1993).
 - [7] T. Damour and B. Pichon, “Big Bang nucleosynthesis and tensor-scalar gravity”, *astro-ph/9807176*.
 - [8] B.J. Carr and S.W. Hawking, *Mon. Not. R. Aston. Soc.* **168**, 399 (1974).